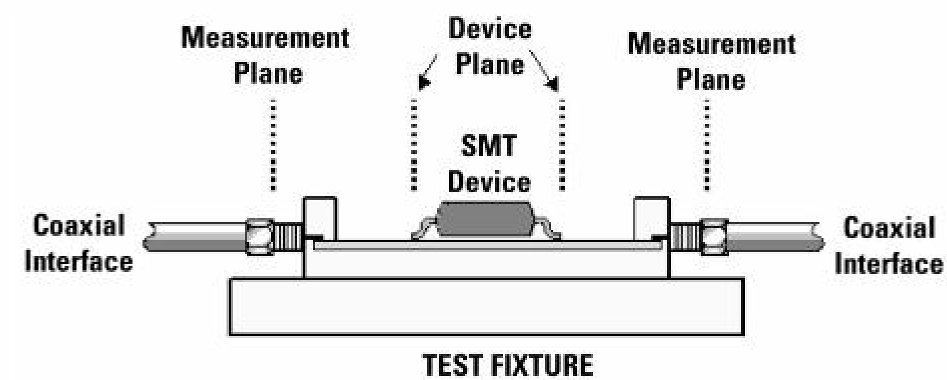
De-embedding and embedding

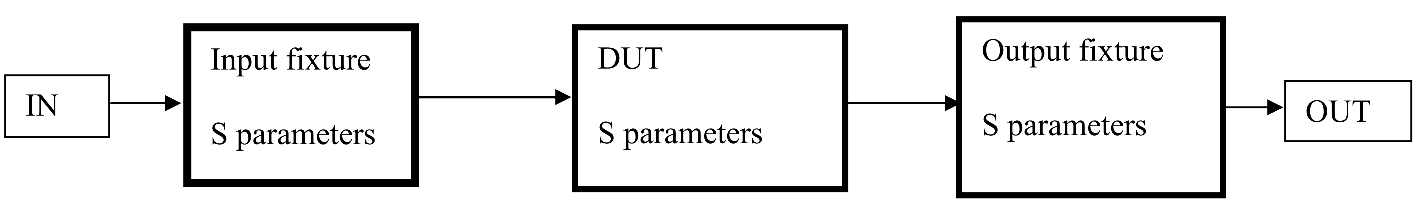
1.0 Introduction: The classic problem is shown below in Figure 1.0.



A SMT device performance (it could be any other type of device) has to be measured with a VNA (for example) whose interfaces are co-axial. To do this the device is mounted (probably) on a substrate and coupled to co-axial interfaces with microstrip and then to the measuring instrument.

De-embedding is a mathematical process that removes the effects of unwanted portions of the structure that are embedded in the measured data by subtracting their contribution.

What can be done is to represent the test fixtures and the DUT by their s – parameters as shown below:



Each of these blocks above are a two port representation in s parameters of the block. (i.e. s11, s12, s21, s22). In order to get the total response of the system we will have to multiply the s parameter matrices.

In order to simplify the task somewhat, it is convenient to use the T – parameter representation of the block. The reason is that when multiplying matrices the T – parameters can be simply cascaded as shown below: (See the definitions of T – parameters in the Appendix).

Therefore

$$\text{Tmeasured} = \text{TIN*TDUT*TOUT} \qquad 1.0$$

Where TIN, TDUT and TOUT are matrices.

This operation represents the T parameters of the test fixtures and the DUT as measured by the VNA.

Tmeasured contains the cascaded data at the *measurement plane.*

Once the data is captured, matrix algebra can be used to de-embed the DUT. This can be done because in matrix algebra an idendity matrix can be found for any non-zero matrix.

Let us find the identity matrix for both the input and output test fixtures. Then:

$$\text{TIN}^{-1} \text{. TIN. TDUT.TOUT.TOUT}^{-1} = \text{TDUT} \qquad 2.0$$

Obviously this operation can be done in postprocessing as well as on some VNAs.

The important operation in addition to just the measurement of the T parameters is the design and construction of the test fixtures. Signal Processing Group Inc., can provide services to develop custom test fixtures or they can purchased commercially if appropriate.

Appendix

The note below defines the T parameters and their relationships to the s – parameters. (From Wikipedia) for convenience.

The Scattering transfer parameters or T-parameters of a 2-port network are expressed by the T-parameter matrix and are closely related to the corresponding S-parameter matrix. The T-parameter matrix is related to the incident and reflected normalized waves at each of the ports as follows:

$$\begin{pmatrix} b_1 \\ a_1 \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$$

However, they could be defined differently, as follows :

$$\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} b_2 \\ a_2 \end{pmatrix}$$

The RF Toolbox add-on to MATLAB and several books (for example "Network scattering parameters") use this last definition, so caution is necessary. The "From S to T" and "From T to S" paragraphs in this article are based on the first definition. Adaptation to the second definition is trivial (interchanging T_{11} for T_{22}, and T_{12} for T_{21}).

The advantage of T-parameters compared to S-parameters is that they may be used to readily determine the effect of cascading 2 or more 2-port networks by simply multiplying the associated individual T-parameter matrices. If the T-parameters of say three different 2-port networks 1, 2 and 3 are (T_1), (T_2) *and* (T_3) *respectively then the T-parameter matrix for the cascade of all three networks (*(T_T)*) in serial order is given by:*

$$(T_T) = (T_1)(T_2)(T_3)$$

As with S-parameters, T-parameters are *complex values* and there is a direct conversion between the two types. Although the cascaded T-parameters is a simple matrix multiplication of the individual T-parameters, the conversion for each network's S-parameters to the corresponding T-parameters and the conversion of the cascaded T-parameters back to the equivalent cascaded S-parameters, which are usually required, is *not trivial*. However once the operation is completed, the complex full wave interactions between all ports in both directions will be taken into account. The following equations will provide conversion between S and T parameters for 2-port networks.

From S to T:

$$T_{11} = \frac{-\det(S)}{S_{21}}$$
$$T_{12} = \frac{S_{11}}{S_{21}}$$
$$T_{21} = \frac{-S_{22}}{S_{21}}$$
$$T_{22} = \frac{1}{S_{21}}$$

From T to S

$$S_{11} = \frac{T_{12}}{T_{22}}$$
$$S_{12} = \frac{\det(T)}{T_{22}}$$
$$S_{21} = \frac{1}{T_{22}}$$
$$S_{22} = \frac{-T_{21}}{T_{22}}$$

Where $\det(S)$ indicates the determinant of the matrix (S).

$$M = \frac{y_{out}}{y_{in}} = \sum_{k=1}^{N} \frac{M_k \Delta_k}{\Delta}$$

Where M is the total gain of the system, represented as the ratio of the output gain (y_{out}) to the input gain (y_{in}) of the system. M_k is the gain of the k^{th} forward path, and Δ_k is the loop gain of the k^{th} loop.