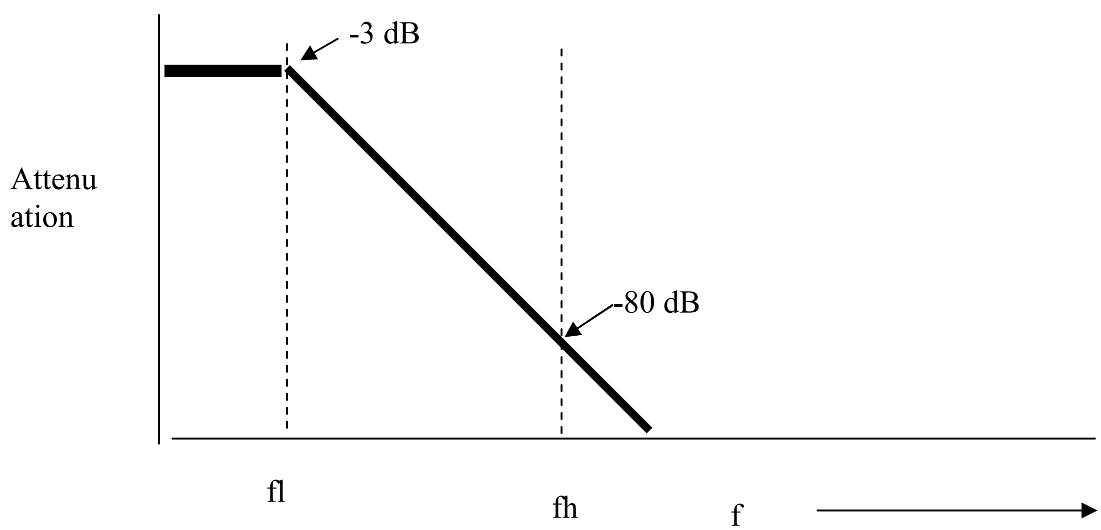
Approximations and filter calculations based on first order filters

1.0 The statement of the problem: (A common problem in ac circuit design)

I wish to find what the -3 dB frequency of a filter should be to attenuate a signal by 80 dB at some high frequency fh. The -3dB frequency can be called fl. Generically the two frequencies would be called fl and fh.
I want use a first order filter whose attenuation is 6 dB/octave.

The figure below illustrates the issue graphically.



How can the solution of this problem be reduced to a technique that allows easy calculations?

To do this, note that a first order filter will attenuate by 6 dB per octave of frequency change. Then lets multiply fl by 2^N which will express the octaves of frequency above fl using fl as the starting point. For example if:

fl =2khz, then 4 octaves above this will be:

f(4 octaves) = 2 * 16 = 32 kHz.

And so on.

Lets take the lower frequency and muliply by 2^N and equate it to the higher frequency, or:

$fl * 2^N = fh$ (1)

then,

$2^N = fh/fl$ (2)

N is the number of octaves between fl and fh. Using the following logarithmic identity we can find N.

$N = \log_{10}(fh/fl)/0.30102$ (3)

$0.30102 = \log_{10}2$

The above logarithmic formula is simply a change of base from 10 to 2.

Once we know how many octaves, we know the attenutation for a first order filter. The same solution should also apply to higher order filters. For example a second order filter will have an atenuation of 12 dB per octave and so on.

Another example:

I want the attenuation at 100 Mhz to be 80 dB. Where should the 3 dB point be using a first order filter.

Solution:

(4)

How many 6 dB's are there in 80 dB?

$N = 80/6 = 13.3$ (5)

Then,

$fl = fh/2^N$ (6)

This gives fl = 10.08E3 Hz. Then our filter should have : (if it is a RC filter)

RC = 1/10.08e3*6.28 (7)

RC = 15.79 microseconds. (8)

Cross check:

1/(6.28*15.79e-6) = 10.08 khz. (approx).

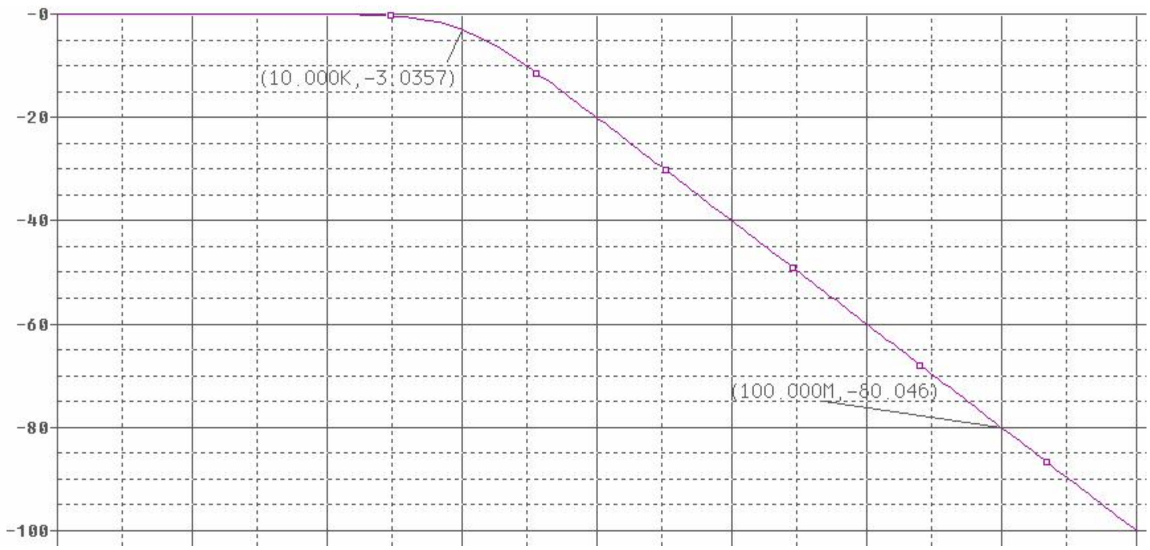
This provids the basic design parameters for the filter. At 100 Mhz we will have a attenuation of:

13.3*6 = 80 dB (9)

These types of problems as well as other related problems may be solved with this simple algorithm.

The figure below shows the simulation results using PSPICE of the filter designed above.

The components used were : R = 10k
C = 1.6 nF



-0
-20
-40
-60
-80
-100
(10.000K,-3.0357)
(100.000M,-80.046)

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