

Expressions for a simple path loss model in free space

For relatively quick estimations of a radio signal's loss in free space one can use the following expressions.

1) Friis free space equation:

$$Pr(d) = [Pt * Gt * Gr * \lambda^2] / (4\pi)^2 * d^2 * L \quad \text{Eqn 1.0}$$

Here:

Pr(d) is the received power ,
Gt, Gr are the transmitter and receiver antenna gains
d is the distance between the transmitter and the receiver
L is a loss factor ($L \geq 1$) of a particular system. $L = 1$
indicates no loss in the system hardware.

2) Antenna gain is given by:

$$G = 4\pi(Ae) / \lambda^2 \quad \text{Eqn 2.0}$$

Here:

Ae is the effective aperture
that is related to the physical size
of the antenna.

3) The wavelength (λ) is:

$$\lambda = c / f = 2\pi c / \omega_c \quad \text{Eqn 3.0}$$

c = speed of light in meters/second
 ω_c = carrier frequency in radians per second.

4) Path loss in dB in free space with antenna gains included is:

$$PL(\text{dB}) = 10\log[Pt/Pr] = -10\log \left[\lambda^2 / (4\pi)^2 * d^2 \right] \quad \text{Eqn 4.0}$$

These equations are valid for the far-field or the Fraunhofer region of the antenna. In this region d, the distance between the transmitter and the receiver should be:

$$d_{fr} > 2D^2 / \lambda \quad \text{Eqn 5.0.}$$

D = largest linear dimension of the antenna

In addition to really be in the far field we must also have:

$$d_{fr} \gg D \text{ and } d_{fr} \gg \lambda \quad \text{Eqn 6.}$$

Obviously this model is invalid for $d = 0$; so choose a close – in distance d_0 as a reference point for received power. The reference point should be in the far – field region and smaller than the distances under consideration for the overall system. Under these assumptions the received power in free space at a distance greater than d_0 is:

$$\begin{aligned} Pr(d) &= Pr(d_0) * (d_0/d)^2 & \text{Eqn 7.0} \\ \text{Here } d &\geq d_0 \geq d_{fr} \end{aligned}$$

Where d_{fr} is the Fraunhofer distance.

Also large changes over many orders magnitude may be expected in received power. So use decibel calculations:

$$\begin{aligned} Pr(\text{dBm}) &= 10\log(P_q) + 20\log(d_0/d) & \text{Eqn 8.0} \\ d &\geq d_0 \geq d_{fr} \\ P_q &= Pr(d_0)/0.001 \text{ Watts} \end{aligned}$$

The voltage at the antenna can be calculated as:

$$V_{\text{antenna}} = \sqrt{Pr(d) * 4 * R_{\text{ant}}} \text{ (rms)} \quad \text{Eqn 9.0}$$

References: ARRL antenna book, Antennas: Krauss, Wireless Communications: Rappaport.