

Re-visiting average and effective (rms) values of voltage, current and power.

Definitions:

For a purely resistive circuit, average power dissipated by a sinusoid is: $P = \frac{1}{2}(V_M \cdot I_M)$ where V_M = peak voltage and I_M is peak current.

For a purely reactive circuit it is: 0!

Given a load Z_L , being supplied by a source with impedance Z_{TH} , maximum average power will be transferred when: $Z_L = Z_{TH}^*$ where Z_{TH}^* is the complex conjugate of Z_L .

If the load is purely resistive then maximum average power will be transferred when: $R_L = \sqrt{R_{TH}^2 + X_{TH}^2}$.

Average and effective values:

Average power absorbed by a resistive load is dependent on the type of source supplying the power.

If the source is DC then average power absorbed is: $I^2 R$
If the source is sinusoidal then average power absorbed is: $\frac{1}{2} (I_M^2)$

These are common waveforms. However most waveforms we encounter will not be so neatly presented. To compare the effectiveness of different sources delivering power to a resistive load an effective value has to be found for comparison.

Define the effective value of a periodic waveform as:

As a constant, or DC value which delivers the same average power to a resistor.

The average power delivered will be: $P = I_{eff}^2 \cdot R$. I_{eff} is also called I_{rms} .

$I_{eff} = \sqrt{\frac{1}{T} \int_{t_0}^{t_0+T} i^2(t) dt}$. The range of integration is from some arbitrary t_0 to t_0+T .

Here T is the time period.

The average power absorbed by a resistor from a periodic waveform is :

$$P = I_{rms}^2 R = V_{rms}^2 / R$$

Note: For example: Normal 120V AC outlets have an rms value of 120V, an average value of 0V, and a maximum value of $120 \cdot 1.41$.

Example:

Find the rms current for a square wave with amplitude varying from -8A to +8A amps with a period of 4usec.

Do the integration:

$$\{1/4e-6(64.dt + 64.dt.)\}^{1/2}$$

The range of integration for the first integral is from 0 to 4usec and the second is from 2usec to 4usec.

This gives: 8A