Transmission line stubs and the design of microwave filters

1.0 Transmission line stubs:

At high frequencies transmission line segments in parallel can be used instead of lumped elements to deliver compatible performance. These segments are frequently referred to as *"open and shorted stubs"*.

Very short line segments terminated in low impedances behave like inductors, while,

very short segments terminated with high impedances behave like capacitors.

Therefore a short circuited (to ground) parallel stub behaves *inductive.* (a short segment here is defined as a segment of 5% to 10% of the effective wavelength)

An open circuited parallel stub behaves like a *capacitor.*

2.0 The mathematics:

Without proof (proof can be found in any EM book)

The input impedance of a transmission line is given by:

$$Zin = Z0 * (ZL + j*Z0*\tan\theta)/(Z0+j*ZL*\tan\theta) \qquad (1)$$

ZL = load impedance
Z0= Characteristic impedance
The other quantities have the usual meanings.

If ZL = 0,

$$Zin = Rshort + jXshort = 0 + j*Zshort*\tan\theta s \qquad (2)$$

i.e. a purely reactive impedance. Zshort is the characteristic impedance of the shorted stub and θs is the electrical length of the stub.

Or the imaginary part simply is the input reactance of the shorted stub.

$$X_{short} = Z_{short} * \tan\theta_s \qquad (3)$$

And the input susceptance is simply its reciprocal.

A similar approach for open circuited stubs yields,

$$X_{open} = Z_{open} / \tan\theta_s \qquad (4)$$

Z_{open} is the characteristic impedance of the open circuited stub, and θ_s is the electrical length.

The susceptance is its reciprocal.

These results apply for stubs with electrical lengths *shorter than 90 degrees.* These results are also valid for *single frequencies* although for a narrow band of frequencies near the frequency of interest the results may be accurate enough for use.

Finally a note on the stubs. *The stubs may look like an inductor or a capacitor depending on their electrical lengths*. Care should be taken to compute the reactances using the equations provided above for accuracy.

3.0   Richard's transformations:

A very useful transformation that helps significantly in the design of microwave filters using transmission line stubs is the Richard's transformation. It transforms inductors and capacitors to transmission line stub quantities so a one to one correspondence may be obtained at a single frequency when transforming lumped L-C filters to transmission line based filters at higher frequencies.

This transformation is described in the following.
Note that the input impedance of a shorted stub may be written as:

$$Z_{in} = j * Z_{short} * \tan\theta_s$$

And the input impedance of an open circuit stub may be written as:

Zin = -j*Zopen*cotθs

Here θs is the electrical length of the stub.

From basic EM theory, the electrical length is also,

βl, where l is the geometric length of the stub.

$$\beta = \omega/v_p \tag{5}$$

Then at one frequency ω,

If inductive reactance,

$$j\omega L = j*Zshort*\tan(\omega l/v_p) \tag{6}$$

or,

$$-j/\omega C = -j*Zopen*\cot(\omega l/v_p) \tag{7}$$

From equation (6) note that equivalence between the inductor value and transmission line can be found by choosing Zshort and $\omega l/v_p$ properly. For example if $\omega l/v_p = \pi/4$ then $\tan(\omega l/v_p) = 1.0$. In such a case,

$$\omega L = Zshort \tag{8}$$

So choose Zshort in such a way that at a singular frequency the reactance of L becomes equal to the characteristic impedance of the shorted stub! Of course the length of line chosen above was 1/8*λ.

A similar procedure can be used for the capacitor. Set the open circuited stub characteristic impedance to be Zopen = 1/ ωC (assuming the same length of line as above). Obviously the length of line can also be another design parameter as needed).

These transformations (at a singular frequency) are called Richard's transformations.

In any case it should be kept in mind that these transformations *do not* generate exact replacements for lumped capacitors and inductors.

An example of the use of these transformations is the conversion of a lumped element low pass filter to a transmission line based filter design. It will be noticed that the behavior of the filter in the stop band is not the same as the lumped element design.

In the passband the match between lumped element design and transmission line design will be close.

The following is an example of design conversion using this method. Figure 1.0 is the filter to be converted.
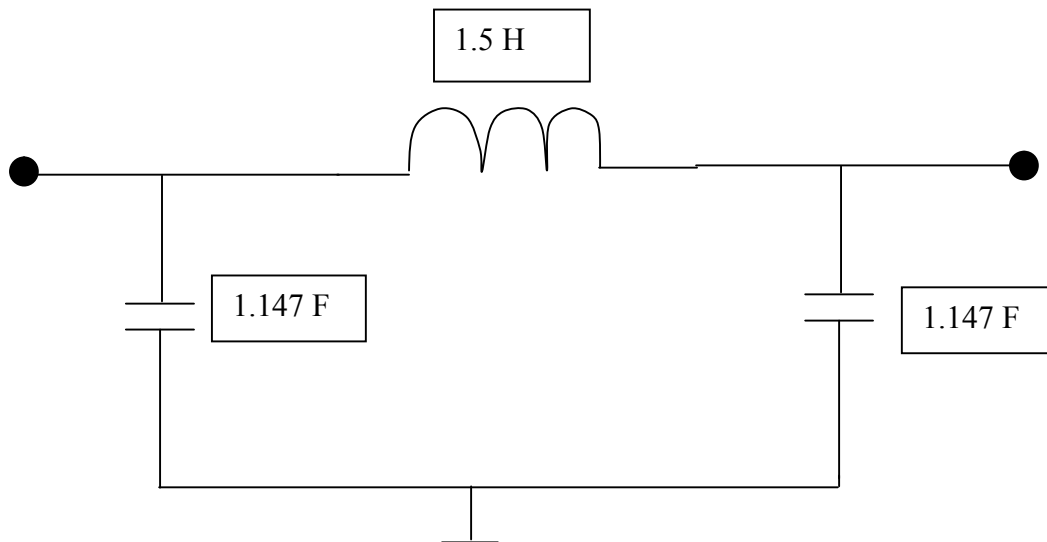

Source and load impedances are 50 Ohm. Two open circuit parallel stubs are required and one short circuited series stub. Set the electrical length arbitrarily to 45 deg at 1 Ghz.

The scaled version of the above circuit is shown below:

3.80e-9

3.65e-12

3.65e-12
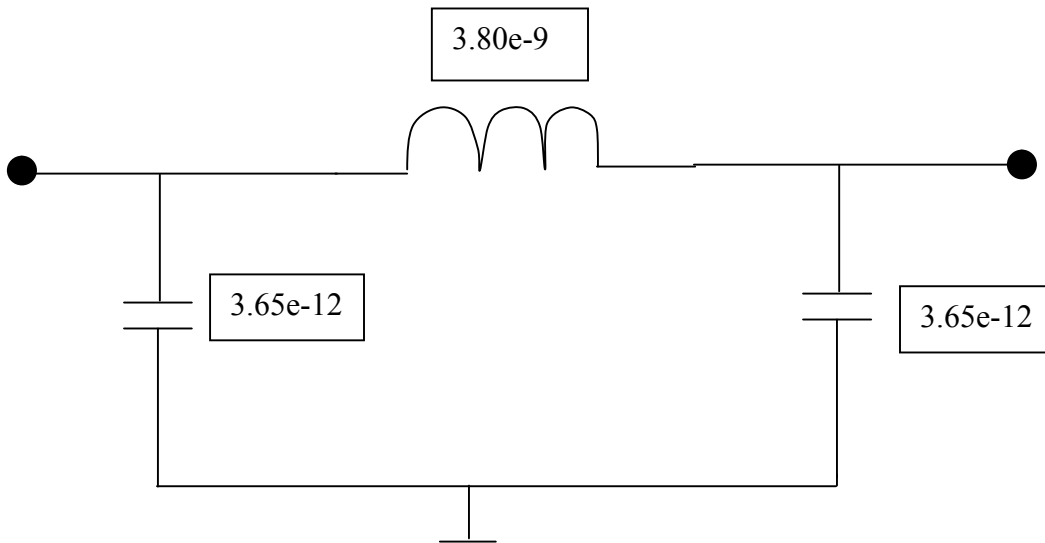
The scaling was done by dividing the inductance and capacitance value by $2.0*\pi*(1e9)$ (the 3dB frequency is 1 Ghz). The magnitude scaling was done by multiplying the inductance by 50 and dividing the capacitance value by 50.

The input and output impedances are 50 Ohm. The simulation results (ac sim) for this circuit are shown below.

Sim circuit
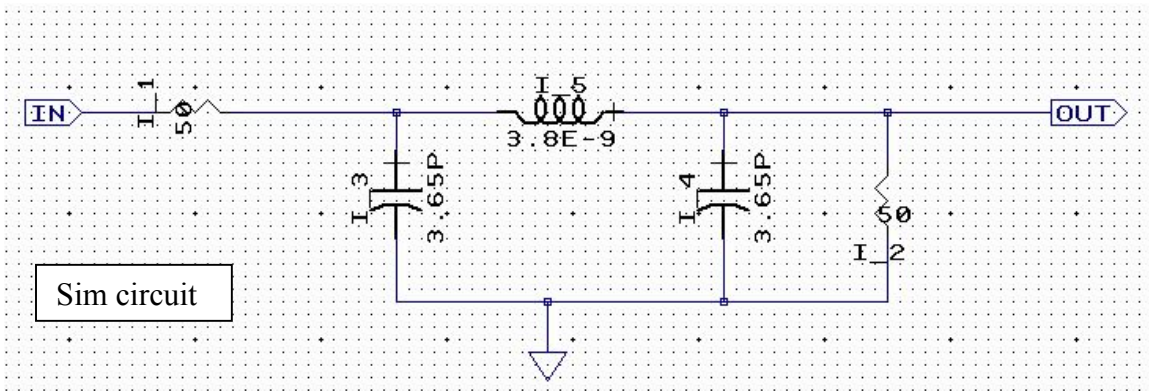
The filter was simulated using PSPICEwithout any optimization or tweaking. (All of that can be done and should be done in a real case. This is Just a demo of the technique).
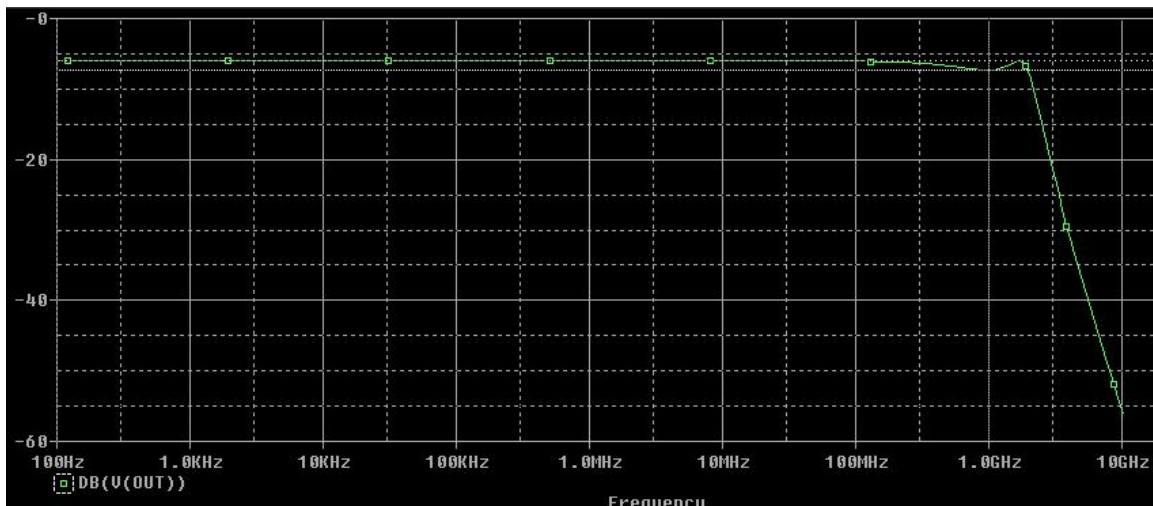
PSPICE simulation (AC) of filter with lumped components.
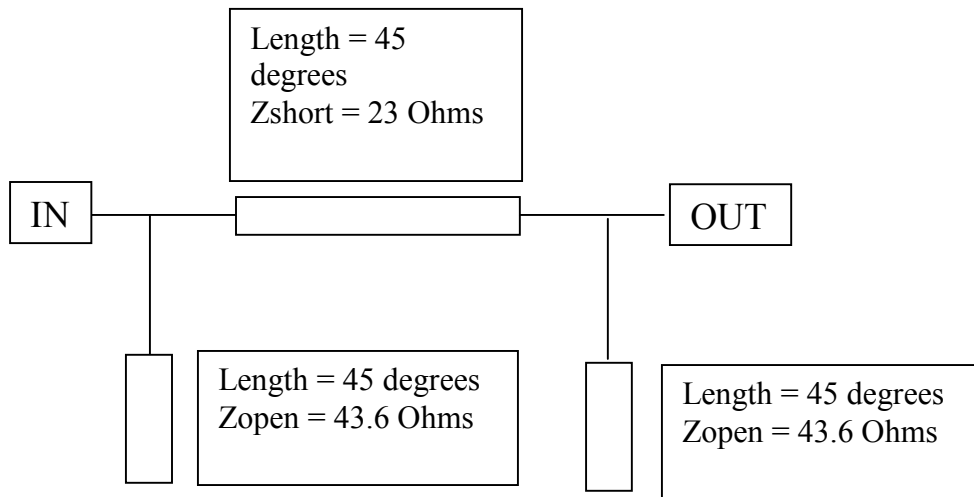
In order to convert the lumped components to transmission line equivalents Richard's transformation is used as shown in the following.

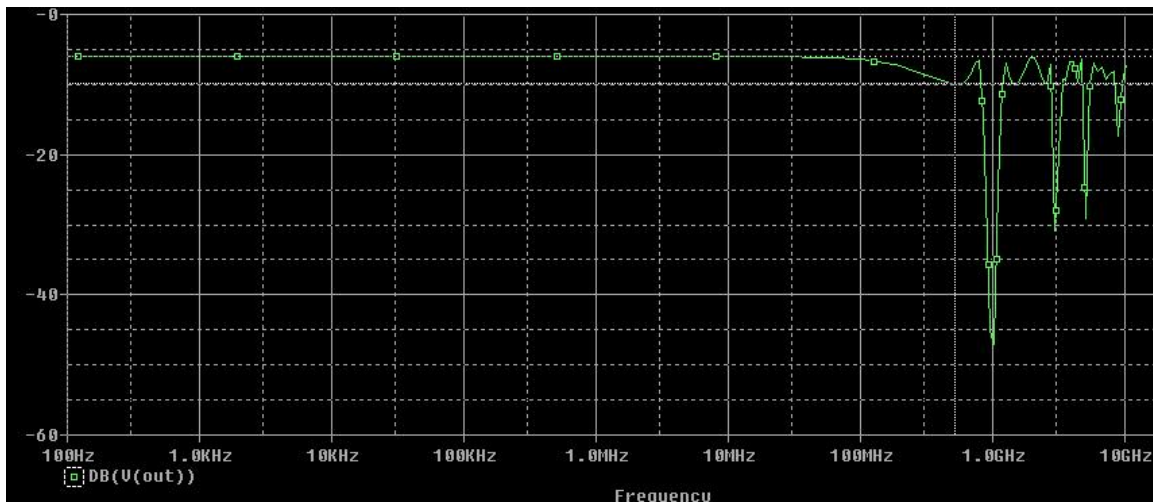At 1 Ghz, calculate the reactance of the inductor: 23 Ohms
At I Ghz calculate the reactance of the capacitor: 43.6 Ohms

Then, use a 45 Degree electrical length of transmission lines and convert the series inductors to series shorted stubs. Then the characterisic impedance of the inductor should be 23 Ohms and the characteristic impedance of the open stub should be 43.6 Ohms.
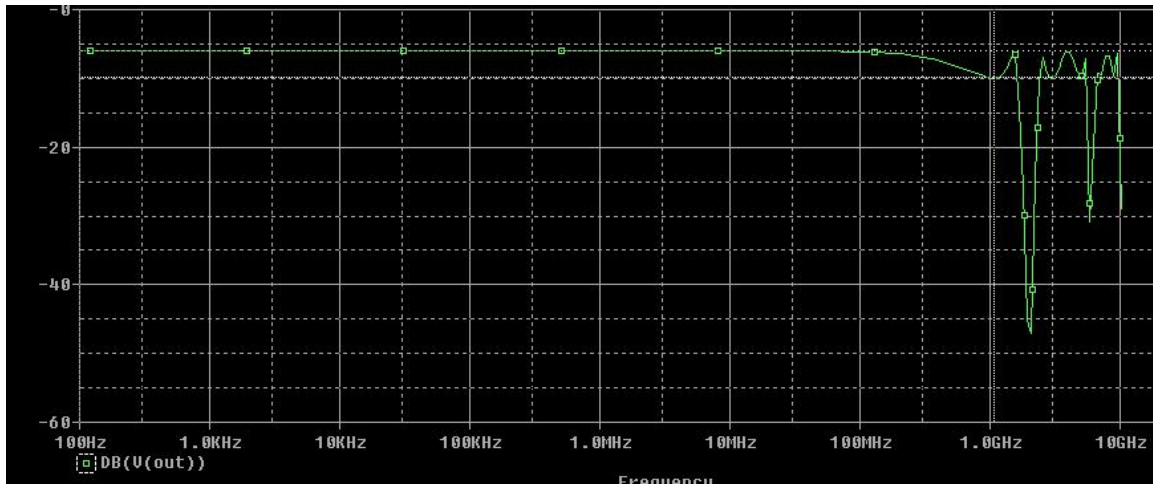
Thus the lumped element circuit changes to the one shown below:

Length = 45
degrees
Zshort = 23 Ohms

IN   OUT

Length = 45 degrees
Zopen = 43.6 Ohms

Length = 45 degrees
Zopen = 43.6 Ohms

Converted transmission line circuit

□ DB(V(out))

Simulation results of the transmission line equivalent circuit. Quarter
wavelength .

The simulated results of the transmission line equivalent circuit is shown above. One eighth wavelength commensurate lengths.

Note how the additional passbands shift with commensurate length reduction.

Some tweaking is obviously required. Also see the notes below:

In the passband the response is very close to the lumped element case. Here is the explanation:

(1) Note that when φ is small, tan(φ) approximately equal to φ. Obviously a similar comment holds true for cot(φ).

(2) The input impedance of the shorted stub (chosen here to illustrate a point) is:

$Zin(short) = jZ0*tan(\beta*\lambda/8)$
Where λ is the wavelength of a signal at some arbitrary frequency ω. The physical length is chosen as λ/8.

This can also be written as:

$Zin(short) = j\omega L*tan[(\omega o/\omega)*(\pi/4)]$ (See definition of β in the blog)

Where ωo is the frequency variable.
However, when ωo<<ω then we can

approximate the above by:
$Zin(short) = j\omega L*[(\omega o/\omega)*(\pi/4)]$
Or,
$Zin(short) = j*\omega o*L[(\pi/4)]$,

Now $\pi/4 = 0.785$, thus we can
Approximate $Zin(short)$ by $j*\omega o*L$

Of course this holds true only when $\omega o \ll \omega$
(Typically the case in a low pass filter passband.

That is why the frequency response of the converted
Filter is very close to the lumped element filter
in the passband.

Comments: Although the response is not very close to the lumped element filter the techniques is quite simple to use and thus could be used effectively given the proper requirements. Other transformations such as Kuroda transformations also exist which can be used to advantage. However, these a the subject of another paper.

This paper brought out a number of interesting points:

(A) The Richard's transformation, a useful transformation in microwave design.
(B) Simulation of transmission line circuits using PSPICE.
(C) Simple conversions of lumped elements circuits to transmission line circuits.